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Signed rank test

The Wilcoxon signed-rank test is a non-parametric statistical hypothesis test used to analyze data and determine if there's a significant difference between two groups. It's often used when the normal distribution of differences cannot be assumed. The test assumes that the distribution of differences is symmetric around a central value, which means it aims to see if this center value differs significantly from zero. This test is more powerful than the sign test and considers the magnitude of differences but requires a stronger assumption of symmetry. The one-sample Wilcoxon signed-rank test is used to determine whether data comes from a symmetric population with a specified center. The test statistic is the signed-rank sum T, which is calculated by summing the product of the sign function (sgn) and the rank of each data point. The ranks are defined such that R_i is the number of j for which |X_j| ≤ |X_i|. The test also considers the positive-rank sum T+ and the negative-rank sum T-. These statistics are related to the signed-rank sum T by equations, showing that any of them can be used as the test statistic. Hypotheses for the Wilcoxon signed-rank sum test include null and alternative hypotheses that involve symmetry around a median or mean. If the null hypothesis states that the distribution is symmetric around zero, one-sided alternatives propose asymmetry either below or above zero. A two-sided alternative suggests non-symmetry around any value. The restriction to symmetric distributions can be relaxed for one-sided tests. Additionally, the test can be applied when each data point comes from a different continuous and symmetric distribution centered at a common median. The paired data test's hypotheses focus on the behavior of differences between paired observations, asserting symmetry or asymmetry around zero. Hypothesis H3 states that the differences between paired observations X and Y are symmetric around zero, meaning that the distribution is the same whether you subtract X from Y or vice versa. There are also alternative hypotheses: * One-sided hypothesis H1: For some μ < 0, the pairs (X, Y) and (Y + μ, X - μ) have the same distribution. * One-sided hypothesis H2: For some μ > 0, the pairs (X, Y) and (Y + μ, X - μ) have the same distribution. * Two-sided hypothesis H3: For some μ ≠ 0, the pairs (X, Y) and (Y + μ, X - μ) have the same distribution. The null hypothesis of exchangeability can occur in a matched pair experiment where treatment and control groups are randomly assigned. This makes the observations exchangeable, meaning that the distribution is symmetric around zero. A paired Wilcoxon test can be used to compare these hypotheses. The test can also be used to compare one-sided alternative hypotheses: * H1: The differences X - Y are stochastically smaller than a distribution symmetric around zero. * H2: The differences X - Y are stochastically larger than a distribution symmetric around zero. In real data, it's common for there to be observations that equal zero or pairs with identical values. This can cause problems when trying to rank the data uniquely. X_{i} is zero and there are no other zeros or ties. This means that the test statistic needs to be modified because Wilcoxon's original paper did not address observations that equal zero. Later surveys recommended removing these zeros from the sample, which would allow the standard signed-rank test to be applied if there were no ties. This is now known as the reduced sample procedure. However, Pratt observed that this procedure can lead to paradoxical behavior. He gave an example of a one-sample situation with 13 observations: 0, 2, 3, 4, 6, 7, 8, 9, 11, 14, 15, 17, and -18. The reduced sample procedure removes the zero and assigns signed ranks to the remaining data: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and -12. This results in a one-sided p-value of 55/2^12, which means that the sample is not significantly positive at any significance level α < 55/2^12 ≈ 0.0134. Pratt argued that decreasing observations should not make the data appear more positive. However, if the zero observation is decreased by an amount less than 2, or if all observations are decreased by an amount less than 1, then the signed ranks become -1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. This results in a one-sided p-value of 109/2^13, which means that the sample would be judged significantly positive at any significance level α > 109/2^13 ≈ 0.0133. The paradox is that decreasing an insignificant sample can cause it to appear significantly positive if α is between 109/2^13 and 55/2^12. To address this issue, Pratt proposed the signed-rank zero procedure. This procedure includes zeros when ranking observations in the sample but excludes them from the test statistic or defines sgn(0) = 0. Pratt proved that the signed-rank zero procedure has several desirable behaviors not shared by the reduced sample procedure. Increasing observed values does not make a significantly positive sample insignificant, and it does not make an insignificant sample significantly negative. If the distribution of observations is symmetric, then the values of μ which the test does not reject form an interval. A sample is significantly positive, not significant, or significantly negative if and only if it is so when zeros are assigned arbitrary non-zero signs, if and only if it is so when zeros are replaced with non-zero values that are smaller in absolute value than any non-zero observation. For a fixed significance threshold α, the probability of calling a set of observations significantly positive (respectively, significantly negative) is a non-decreasing (respectively, non-increasing) function of μ. When the signed-rank zero procedure is combined with the average rank procedure, the resulting test is a consistent test against the alternative hypothesis. According to Pratt, this test assumes that Pr (X i + X j > 0) {\displaystyle \Pr(X_{i}+X_{j}>0)} and Pr (X i + X j < 0) {\displaystyle \Pr(X_{i}+X_{j}